Zero-Inflated Tweedie Boosted Trees with `CatBoost` for Insurance Loss Analytics

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Introduction

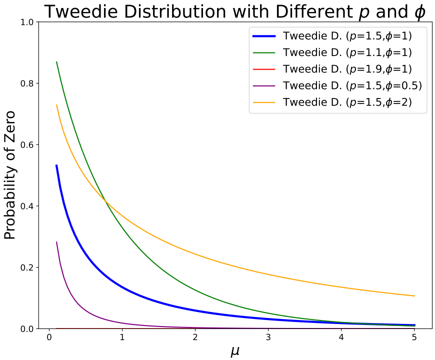
- The two-part frequency-severity models have historically been the norm.
- Since Tweedie et al. (1984), the Tweedie distribution has gained popularity as it eliminates need for separate frequency and severity models.
- Tweedie models, denoted as $\mathrm{Tw}(\mu, \phi, p)$, are defined by the following density function:

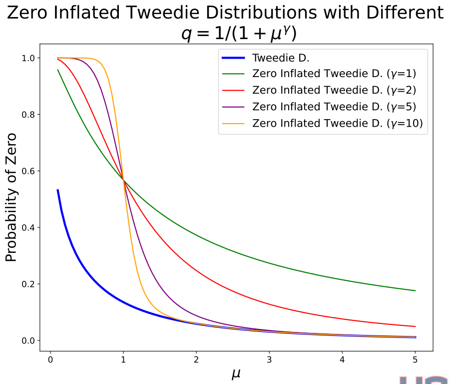
$$f_{\mathrm{Tw}}(y|\mu,\phi,p) = a(y,\phi,p) \exp\left(\frac{1}{\phi}\left(y\frac{\mu^{1-p}}{1-p} - \frac{\mu^{2-p}}{2-p}\right)\right), \quad y \geq 0,$$

  where $a(\cdot)$ is normalizing function, $\mu > 0$ is the expected value of Y, and $\phi > 0$ represents the dispersion parameter.
- $\mathrm{Var}(Y) = \phi\mu^p$ so that p controls the relationship between variance and mean.
- We restrict the power p to $1 < p < 2$, the case of the compound Poisson-gamma model.
- When introducing predictor variables, we can consider using suitable link function.

Zero-inflation

- Tweedie distribution is largely flexible and is able to model a wide range of data including those with excess zeros (zero-inflation), right-skewness, and heavy tails, but ...





Zero-inflated Tweedie (ZITw) Distribution Model

- The ZITw model combines a point mass at zero, to help improve the accuracy of estimating μ especially when dealing with excessive zeros.
- The density function of the ZITw model can be formulated as follows:

$$f_{\text{ZITw}}(y|\mu,\phi,p,q) = \begin{cases} q + (1-q)\cdot\exp\left(-\frac{1}{\phi}\frac{\mu^{2-p}}{2-p}\right), & \text{if } y = 0 \\ (1-q)\cdot a(y,\phi,p)\cdot\exp\left(\frac{1}{\phi}\left(y\cdot\frac{\mu^{1-p}}{1-p} - \frac{\mu^{2-p}}{2-p}\right)\right), & \text{if } y > 0. \end{cases}$$

- q represents the inflation probability, indicating the degree of zero inflation.
- The expected value of Y under the ZITw model is given by $(1-q)\mu$. Thus, accurately estimating both μ and q is crucial.
- The gradient boosting framework offers techniques to achieve this effectively.

Gradient Boosting

- Gradient boosting is an ensemble technique based on concept of building a strong predictive model by combining the predictions of multiple weak learners. Friedman (2001).
- When decision trees are used as weak learners, they are called Gradient Boosted Decision Trees (GBDT).
- Given training dataset $\mathcal{D} = \{(\boldsymbol{x}_i, y_i)\}_1^n$, gradient boosting generates a sequence of functions $W_0, W_1, \cdots, W_T$, by minimizing the exp. value of a specified loss function, $\ell(y_i, W_t)$.
  - Each iteration trains a new weak learner to correct ensemble errors.
  - At each iteration, calculate the negative gradient (pseudo-residuals) of the loss function, which gives direction of steepest descent to minimize loss.
  - Newly trained weak learner is fitted to the pseudo-residuals; it learns to predict the errors made by the current ensemble.
  - Update ensemble by adding output to the current ensemble, with a learning rate.
  - Process is repeated for a fixed number of iterations.
  - Final prediction is the cumulative result of all weak learners combined.

Zero-inflated Tweedie Boosted Decision Trees

- We use the negative log-likelihood of the data based on the zero-inflated Tweedie distribution model.
- We use decision trees as weak learners.
- The boosted tree model assumes the logarithm of the exp. value of target variable Y, given set of features $\boldsymbol{x}$, can be effectively modeled as follows:

$$\ln \mathbb{E}(Y \mid \boldsymbol{x}) = \ln E + W_T(\boldsymbol{x}),$$

where $\ln E$ is the offset term and $W_T(\boldsymbol{x})$ denotes the prediction score produced as:

$$W_T(\boldsymbol{x}) = w_1(\boldsymbol{x}) + w_2(\boldsymbol{x}) + \cdots + w_t(\boldsymbol{x}) + \cdots + w_T(\boldsymbol{x}).$$

- Here, $w_t(\boldsymbol{x})$ represents the prediction of the t-th tree in the gradient boosting model.
- This framework allows for a flexible and powerful modeling of complex relationships between features and target variable.

Categorical Boosting (CatBoost)

- Notable software libraries for GBDT implementation include XGBoost, LightGBM, and CatBoost.
- Increasing in popularity, CatBoost, developed by Yandex (Prokhorenkova et al., 2018), is recognized for its effectiveness in handling heterogeneous datasets, a common scenario in insurance data.
- It employs a technique known as "Ordered Target Statistic" in encoding categorical features as numerical features.
- Additional advantages include: producing high predictive accuracy, offering scalability for large data sets, and supporting the generation of interpretative graphs that help in further understanding and explaining model results.
- Recent studies (So, 2024) have demonstrated CatBoost's superior performance compared to its counterparts when processing insurance data.

Methodology

- In conventional zero-inflated models, training is usually conducted separately for the mean μ and the inflation probability q.
- This approach requires twice as many trees for zero-inflated Tweedie (ZITw) boosted trees compared to Tweedie (Tw) models, due to independent parameter estimation for each:

$$\ln \mu = \ln E + W_T^{mean}(\boldsymbol{x}),$$

$$\text{logit}(q) = \ln \frac{q}{1-q} = W_T^{prob}(\boldsymbol{x}).$$

Two possible approaches

- **Scenario 1** Functionally unrelated: q is not directly functionally related to μ
  - Train $W_T^{mean}(\boldsymbol{x})$ and $W_T^{prob}(\boldsymbol{x})$ separately.
- **Scenario 2** Functionally related: q is functionally linked to μ
  - Our proposed parameterization is depicted by the following equations:

$$\ln \mu = \ln E + W_T(\boldsymbol{x}),$$

$$\text{logit}(q) = \ln \frac{q}{1-q} = -\gamma(\ln E + W_T(\boldsymbol{x})).$$

This leads us to $q = \frac{1}{1+\mu^\gamma}$.

Adjustment of Compositional Data

- Compositional data is characterized by multiple non-negative features that sum up to a constant, typically 100% or 1.
- Due to the inherent statistical dependence among these features, transformations are often necessary to map the data onto the real Euclidean space.
- This transformation facilitates the application of traditional statistical methodologies.
- When dealing with compositional data comprising J features, denoted as $\{\boldsymbol{x}_{\cdot 1}, \boldsymbol{x}_{\cdot 2}, \ldots, \boldsymbol{x}_{\cdot J}\}$, where the features sum to 1, we refer to these features as a J-part composition.
- See Aitchison (1994)

Logratio transformations

- Notable transformations are the logratio methods, which include:
  - centered logratio transformation (CLR):

$$\mathrm{CLR}(j) = \ln\left(\frac{\boldsymbol{x}_{\cdot j}}{\left(\prod_i \boldsymbol{x}_{\cdot i}\right)^{1/J}}\right), \quad j = 1, 2, \ldots, J.$$

  - additive logratio transformation (ALR):

$$\mathrm{ALR}(j|d) = \ln\left(\frac{\boldsymbol{x}_{\cdot j}}{\boldsymbol{x}_{\cdot d}}\right), \quad j \neq d.$$

  - isometric logratio transformation (ILR):

$$\mathrm{ILR}(\boldsymbol{x}) = R \cdot \mathrm{CLR}(\boldsymbol{x}),$$

    where $\boldsymbol{x}$ is a $J \times n$ data matrix comprising J features, and R is a $(J-1) \times J$ matrix satisfying the condition: $RR^T = I_{J-1}$.

Models Compared

Our empirical analysis is based on a synthetic telematics dataset developed by So et al. (2021). This dataset comprises of 100,000 policies and demonstrates a zero-inflation characteristic, with only 2,698 policies experiencing at least one claim. For this study, a total of eight different models were trained:

1. Zero-inflated Tweedie boosted tree with scenario 1 (ZITwBT1)
2. Zero-inflated Tweedie boosted tree with scenario 2 (ZITwBT2)
3. Tweedie boosted tree (TwBT)
4. Tweedie GLM (TwGLM)
5. ALR
6. CLR
7. ILR
8. PPCA after CLR transformation

Performance Metrics

- Deviance: measures how well the predicted outcomes in a model match the observed outcomes. Lower deviance indicates better fit.
- Mean Absolute Deviation: quantifies the average absolute difference between the observed and predicted values, defined as $\text{MAD} = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i|$. A lower MAD suggests higher precision.
- Vuong Test: compares likelihood functions of non-nested models.
- Gini Index: assesses model prediction performance. Gini^a and Gini^b are two variants.

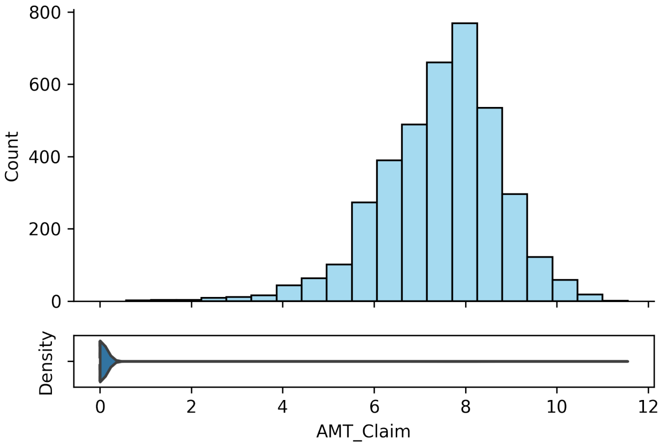
Descriptive Details of Dataset

Table 1: Variable Names and Descriptions for the Synthetic Telematics Dataset

Type	Variable	Description
Traditional	Duration	Total exposure in yearly units
	Insured.age	Age of insured driver
	Insured.sex †	Sex of insured driver: Male, Female
	Car.age	Age of vehicle (in years)
	Marital †	Marital status: Single, Married
	Car.use †	Use of vehicle: Private, Commute, Farmer, Commercial
	Credit.score	Credit score of insured driver
	Region †	Type of region where driver lives: Rural, Urban
	Annual.miles.drive	Annual miles expected to be driven declared by driver
	Years.noclaims	Number of years without any claims
	Territory †	Territorial location of vehicle: 55 labels in {11, 12, 13, . . ., 91}
Telematics	Annual.pct.driven	Annualized percentage of time on the road
	Total.miles.driven	Total distance driven in miles
	Pct.drive.xxx	Percent of driving day xxx of the week: mon/tue/. . . /sun
	Pct.drive.x hrs	Percent vehicle driven within x hrs: 2hrs/3hrs/4hrs
	Pct.drive.xxx	Percent vehicle driven during xxx: wkday/wkend
	Pct.drive.rush xx	Percent of driving during xx rush hours: am/pm
	Avgdays.week	Mean number of days used per week
	Accel.xxmiles	Number of sudden acceleration 6/8/9/. . . /14 mph/s per 1000miles
	Brake.xxmiles	Number of sudden brakes 6/8/9/. . . /14 mph/s per 1000miles
	Left.turn.intensityxx	Number of left turn per 1000miles with intensity xx: 08/09/10/11/12
	Right.turn.intensityxx	Number of right turn per 1000miles with intensity xx: 08/09/10/11/12
Response	NB_Claim	Number of claims on the given policy
	AMT_Claim	Amount of claims on the given policy

† Indicates categorical variable.

Distribution of Aggregate Claim Amounts



Model Goodness of Fit

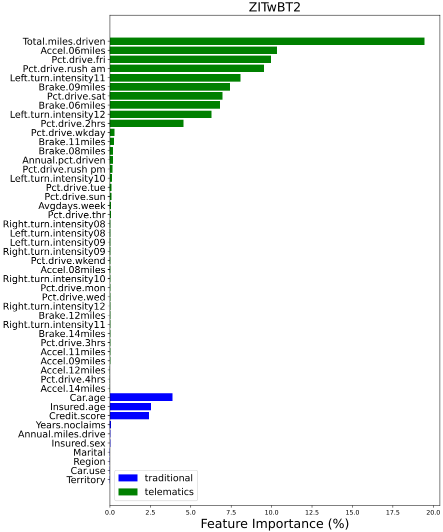
Table 2: Gini[b] across 4 Models

Base Model \ Competing Model	TwGLM	TwBT	ZITwBT1	ZITwBT2
TwGLM	-	0.489	0.120	0.504
TwBT	-0.043	-	-0.275	0.266
ZITwBT1	0.695	0.598	-	0.704
ZITwBT2	0.127	0.035	-0.105	-

Table 3: Gini[b] in ZITwBT2 Models with and without Compositional Data Adjustment

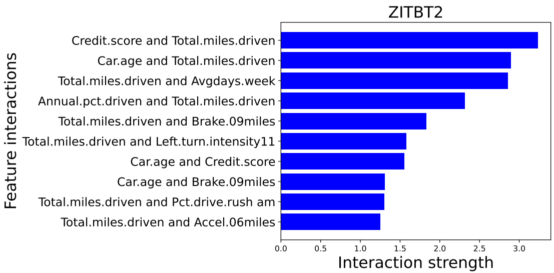
Base Model \ Competing Model	ZITwBT2	ZITwBT2 alr	ZITwBT2 clr	ZITwBT2 ilr	ZITwBT2 clr+PPCA
ZITwBT2	-	0.128	0.122	0.124	0.011
ZITwBT2 alr	0.160	-	0.145	0.119	0.033
ZITwBT2 clr	0.762	0.760	-	0.755	0.735
ZITwBT2 ilr	0.152	0.167	0.138	-	0.059
ZITwBT2 clr+PPCA	0.335	0.349	0.342	0.304	-

Feature importance in ZITwBT2 CatBoost

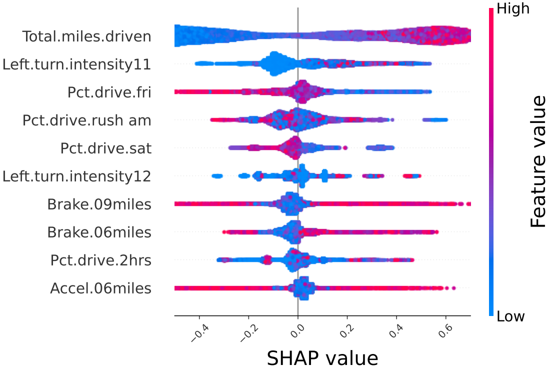


- More telematics than traditional variables are better predictors of aggregate claims.
- `Total.miles.driven` far outweigh all other variables.
- Driving maneuvers appear to be important predictors of aggregate claims.

Features Interaction and SHAP Values in ZITwBT2 CatBoost

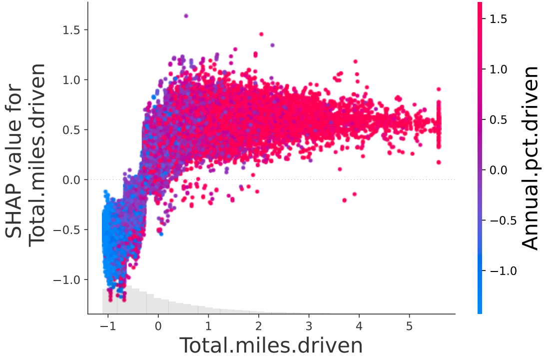


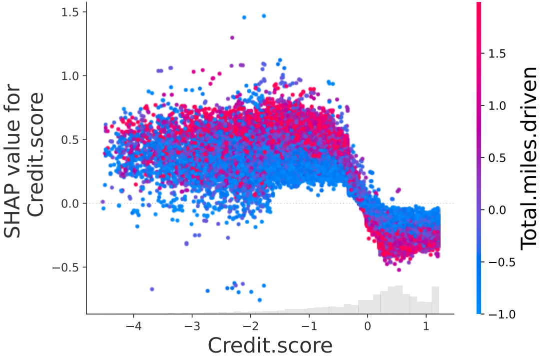
Top 10 Features Interaction Strength



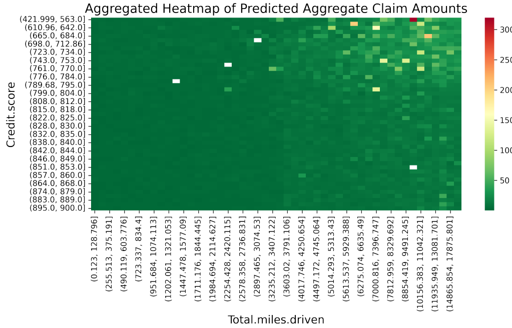
SHAP Values of Top Important Features

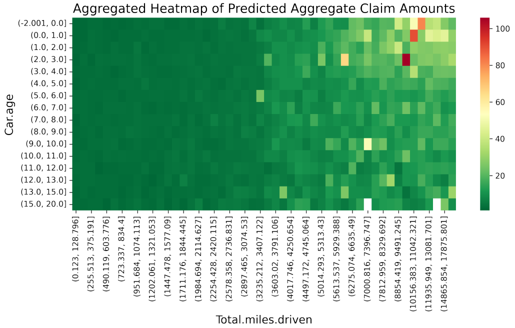
Selected Feature Interaction Through SHAP Values





Heatmaps Describing Feature Interaction with Aggregate Claim Amount





Concluding remarks

- In this paper, we applied a zero-inflated Tweedie loss function in gradient boosting with various adjustments.
  - We reparameterized the zero-inflated Tweedie loss function to express the inflation probability q as a function of μ.
  - This reparameterization led to a unified model, maximizing the use of `CatBoost` libraries.
  - This approach improves interpretation and enables better model comparison through various performance metrics.
- Our research makes significant contributions to actuarial studies as a result of:
  - Simplified interpretation and efficiency
  - Robustness to compositional data. Our model shows robustness without needing extra adjustments for compositional data, indicating superiority over GLMs.
  - Advantages of using `CatBoost` libraries.